Linear Time-Invariant Systems

Modules: Wideband True RMS Meter, Audio Oscillator, Utilities, Digital Utilities, Twin Pulse Generator, Tuneable LPF, 100-kHz Channel Filters, Phase Shifter, Quadrature Phase Splitter, Baseband Channel Filters, Noise Generator

0 Pre-Laboratory Reading

0.1 Frequency Response of a Filter

A filter is a linear time-invariant (LTI) system. The response of a filter to a sinusoid is a sinusoid of the same frequency. (If the filter is designed to block that frequency, the response can be zero.) The amplitude and the phase of the sinusoid will, in general, have changed in passing through the filter. The (complex-valued) frequency response $H(f)$ of the filter accounts for these changes.

The magnitude $|H(f)|$ of the frequency response is the factor by which the amplitude changes, and this factor is a function of the frequency f of the sinusoid. The phase angle $\arg H(f)$ of the frequency response is the change in the phase (with units radians) of the sinusoid, and it too is a function of f.

$$\sqrt{2}\cos(2\pi ft) \longrightarrow \boxed{\text{Filter}} \longrightarrow \sqrt{2}|H(f)|\cos[2\pi ft + \arg H(f)]$$

The sinusoid is delayed in going through the filter, so $\arg H(f)$ is normally negative for positive values of f. The time delay τ, in seconds, is defined to be a positive quantity. The output of the filter can also be written in terms of τ, and this looks like: $\sqrt{2}|H(f)|\cos[2\pi f(t - \tau)]$. The phase $\arg H(f)$ is related to the time delay by

$$\arg H(f) = -2\pi f\tau \tag{1}$$

0.2 Train of Uniformly Spaced, Narrow Pulses

A Dirac comb in the time domain is a uniformly spaced set of equal-weight Dirac delta functions. Its Fourier transform is a Dirac comb in the frequency domain.

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \leftrightarrow \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right) \tag{2}$$
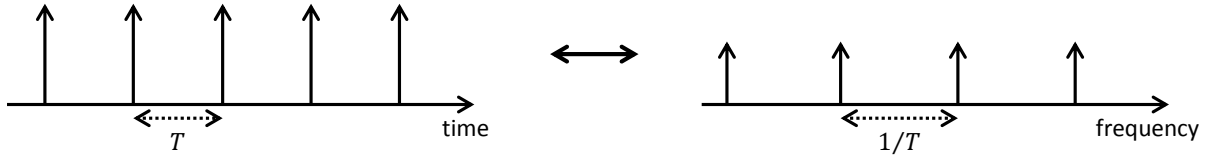
The spacing in the time domain is T seconds, and the spacing in frequency domain is $1/T$ Hz.

The Fourier transform of a sinusoid is one Dirac delta in the positive half of the frequency axis (and a second Dirac delta symmetrically located in the negative half of the frequency axis). A set of equally-weighted Dirac deltas, uniformly spaced by $1/T$ in the frequency domain, must therefore be the Fourier transform of a set of equal-amplitude sinusoids whose frequencies are all integer multiples of one fundamental frequency, $1/T$. We have identified two functions of time that have the same Fourier transform. These two functions of time (signals) must therefore be equivalent. In other words, a Dirac comb in the time domain is equivalent to a set of sinusoids that are uniformly spaced in frequency and of equal amplitude.
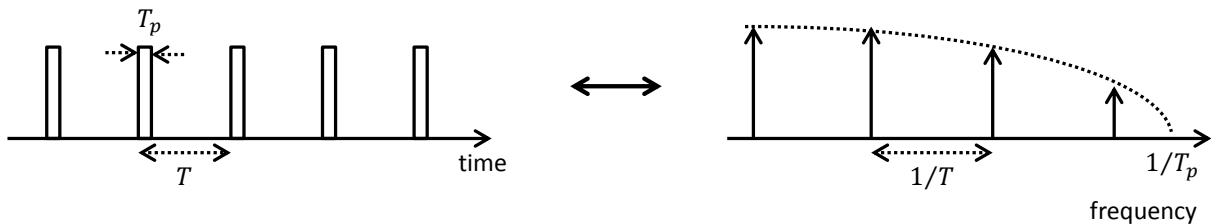
A train of uniformly spaced, narrow pulses is related to the Dirac comb. If $p(t)$ is an isolated, narrow rectangular pulse, then $\sum_{n=-\infty}^{\infty} p(t - nT)$ is a train of such pulses, uniformly spaced and of equal amplitude. This pulse train can also be represented as the convolution of $p(t)$ with a Dirac comb.

$$\sum_{n=-\infty}^{\infty} p(t - nT) = p(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT) \tag{3}$$

From Equation (3) and the Convolution Theorem, the Fourier transform of a train of uniformly spaced, narrow pulses equals the product of the Fourier transform $P(f)$ of $p(t)$ and the Fourier transform of a Dirac comb.

$$\sum_{n=-\infty}^{\infty} p(t - nT) \leftrightarrow P(f) \cdot \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right) \tag{4}$$

The magnitude of the Fourier transform $P(f)$ of a rectangular pulse equals the absolute value of a sinc function. The first null of $P(f)$ occurs at $f = 1/T_p$ Hz, where T_p is the width of the pulse.


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In summary, the Fourier transform of a train of uniformly spaced, equal-amplitude, narrow and rectangular pulses is a set of uniformly spaced Dirac deltas in the frequency domain, but these Dirac deltas are not of equal weight. However, if the pulse width T_p is much less than the spacing T between pulses, then the Dirac deltas located at frequencies f within the range $0 \leq |f| \leq 0.4/T_p$ are of approximately the same weight. (The weight of the Dirac delta at $f = 0.4/T_p$ is 76% of the weight of the Dirac delta at $f = 0$.) Hence, a train of uniformly spaced, equal-amplitude, narrow and rectangular pulses is equivalent to a set of sinusoids spaced equally in frequency, and the amplitudes of those sinusoids are approximately equal for frequencies f within the range $0 \leq f \leq 0.4/T_p$, where T_p is the pulse width.

0.3 Approximation of a Square-Wave by Some of Its Harmonics

A square-wave possesses a Fourier series expansion. For a square-wave of zero average value with an amplitude of A, the magnitudes of the Fourier series coefficients are

$$
|a_k| = \begin{cases} \dfrac{2A}{k\pi}, & k \text{ odd} \\[2mm] 0, & k \text{ even} \end{cases} \tag{5}
$$

The square-wave may be approximated with only the fundamental and just some of the harmonics. (The k-th harmonic consists of two complex exponentials: $a_k e^{j2\pi kt/T}$ and $a_{-k} e^{-j2\pi kt/T}$, where T is the period of the square-wave.) The figure below illustrates this. The curve on the left shows a (poor) approximation consisting of just the fundamental. The middle curve includes the first and third harmonics. The curve on the right includes the first, third and fifth harmonics. The more harmonics that are included, the better the approximation.

First harmonic only   First and third harmonics   First, third and fifth harmonics
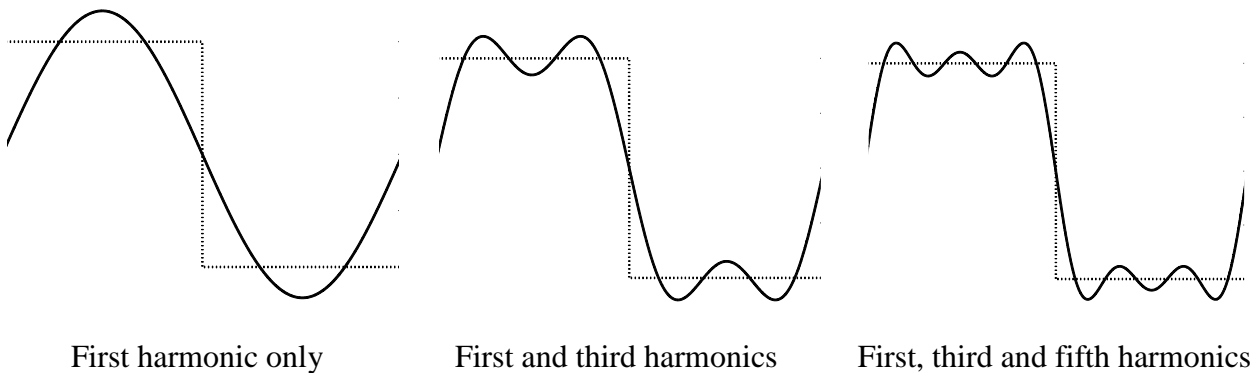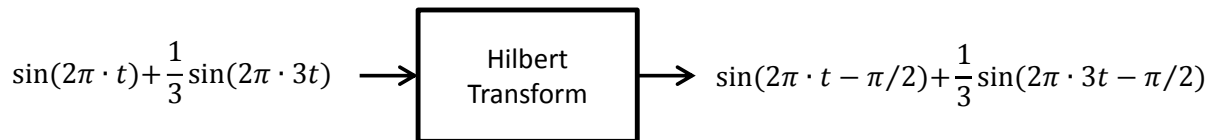
0.4 Hilbert Transform

For a Hilbert transform, both the input and the output are in the time domain. This is unlike the Fourier transform, for which the input is in the time domain and the output is the frequency domain description of the input. The Hilbert transform is a linear, time-invariant system. If the input is a sinusoid, the output is also a sinusoid of the same frequency. Here is an example:

$$\cos(2\pi \cdot 5t) \longrightarrow \boxed{\begin{array}{c}\text{Hilbert} \\ \text{Transform}\end{array}} \longrightarrow \cos(2\pi \cdot 5t - \pi/2)$$

For a sinusoidal input, the output has a phase that is less than that of the input by $\pi/2$ radians. The amplitude is unchanged between input and output.

Here is a more complicated example:

$$\sin(2\pi \cdot t) + \frac{1}{3}\sin(2\pi \cdot 3t) \longrightarrow \boxed{\begin{array}{c}\text{Hilbert} \\ \text{Transform}\end{array}} \longrightarrow \sin(2\pi \cdot t - \pi/2) + \frac{1}{3}\sin(2\pi \cdot 3t - \pi/2)$$

In this example, each sinusoid on the input gives rise to a sinusoid of the same frequency on the output. Each output sinusoid has a phase that is less than that of its corresponding input by $\pi/2$ radians. The amplitude of each sinusoid is unchanged between input and output.

This is not the same as a time delay. A time delay of τ is equivalent to a phase change of $-2\pi f\tau$, where f is the frequency of the sinusoid. For example, if the input shown above were delayed by $\tau = 1/4$ second, the result would be

$$\sin\left[2\pi\left(t - \frac{1}{4}\right)\right] + \frac{1}{3}\sin\left[2\pi \cdot 3\left(t - \frac{1}{4}\right)\right] = \sin\left[2\pi \cdot t - \frac{\pi}{2}\right] + \frac{1}{3}\sin\left[2\pi \cdot 3t - \frac{3\pi}{2}\right] \qquad (1)$$

This is different from the output of the Hilbert transform. A delay of $\tau = 1/4$ second corresponds to a loss in phase of $\pi/2$ radians for a sinusoid of frequency 1 Hz but a loss in phase of $3\pi/2$ radians for a sinusoid of frequency 3 Hz.

The two-sinusoid input considered above and its Hilbert transform are shown on the next page.
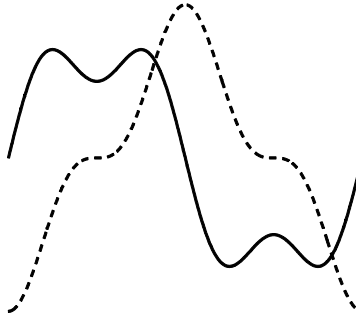
$\sin(2\pi \cdot t) + \frac{1}{3}\sin(2\pi \cdot 3t)$ (solid curve) and its Hilbert transform (dashed curve)

The Hilbert transform of a signal does not, in general, look like the original signal. (In the exceptional case of a single sinusoid, however, the Hilbert transform does look like a time-delayed version of the original.) In the above example, the input signal is the first two terms in the Fourier series expansion of a square-wave; the Hilbert transform looks quite different.

1 Frequency Response of a Filter

You will measure both the magnitude and the phase angle of an example filter's frequency response. Then you will display the magnitude of a frequency response on the PicoScope (Spectrum Mode).

1.1 Measurement of Magnitude and Phase Angle of a Frequency Response

You will measure the frequency response, both magnitude and phase angle, of a low-pass filter. For this purpose you will use a Tuneable LPF. This is a low-pass filter with an adjustable bandwidth and an adjustable passband gain. As a first step, you will adjust the Tuneable LPF for a bandwidth of 5 kHz and a DC voltage gain of 1.

You can adjust the bandwidth of the Tuneable LPF by connecting the (TTL) clock output of this module to the TTL input of the Frequency Counter and adjusting the tune control of the Tuneable LPF. The frequency you measure is 100 times the bandwidth of the Tuneable LPF. To set the bandwidth to 5 kHz, you will first set the toggle switch on the front of the Tuneable LPF to *norm* and then adjust the tune control of the Tuneable LPF until the Frequency Counter indicates 500 kHz. When you have finished setting the bandwidth of the Tuneable LPF, you should remove the patch cord from that module's TTL clock output; leaving a cord connected to that particular clock output causes a lot of noise to be added to the signal passing through the filter.

Place the Variable DC signal at the input of the Tuneable LPF. First connect the RMS Meter to the filter input and adjust the knob on the Variable DC panel until a voltage of 1.00 V rms is indicated. Then connect the RMS Meter to the filter output and adjust the gain control of the

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filter until a voltage of 1.00 V rms is indicated. The DC voltage gain of the filter has now been set equal to 1.

Connect the Audio Oscillator output to both the input of a Buffer Amplifier and to the Frequency Counter. (If you have not already done so, you should disconnect the Tuneable LPF clock signal from the TTL input of the Frequency Counter, as this meter can only measure one frequency at a time.) Remove the DC signal from the filter, and connect the Buffer Amplifier output to the filter input. Place the filter input on Channel A and the filter output on Channel B.

Tune the Audio Oscillator frequency to approximately 1 kHz. Connect the RMS Meter first to the filter input, and adjust the amplifier gain for a filter input voltage of 1.00 V rms. Then connect the RMS Meter to the filter output, and record the rms voltage. Since the filter input is 1.00 V rms, the magnitude of the frequency response equals the rms voltage on the filter output.

Channel A: LPF input
Channel B: LPF output

Now measure the filter's time delay. Use Channel A (the filter input) as the trigger source, and make sure the oscilloscope is set to trigger on positive-going zero-crossings. You should notice that the Channel A sinusoid has a positive-going zero-crossing in the center of the display (assuming the pre-triggering percentage has the default value of 50%), at time equals 0, and that this zero-crossing is marked as the trigger point. The sinusoid on Channel B (the filter output) will be delayed relative to the Channel A sinusoid. The filter delay equals the time span between the trigger point (at time 0) and the Channel B sinusoid's first positive-going zero-crossing to the right of the trigger point. Use a time ruler to accurately measure this delay.

Repeat the above procedure for measuring the voltage gain and the delay of the filter for a set of frequencies. Because you are using an input voltage of 1 V rms, $|H(f)|$ equals the measured rms voltage of the filter output.

frequency response of a filter

The time delay τ, considered a positive quantity, is measured on the oscilloscope (as explained above). The phase delay, $\arg H(f)$, is considered negative and is calculated from the time delay τ as $-2\pi f\tau$, where f is the actual frequency. In the case of a DC signal, the concept of delay is not applicable (NA).

You need not measure τ and you need not calculate $\arg H(f)$ for $f \geq 5$ kHz. Within the passband of the filter, it is desirable for $\arg H(f)$ to be an approximately linear function of f; in other words, τ should be approximately constant (independent of f). Outside the passband, the value of $\arg H(f)$ is unimportant.

Plot $|H(f)|$ as a function of f ($0 \leq f \leq 6$ kHz) using discrete data points. Plot $\arg H(f)$ as a function of f ($1 \leq f \leq 4$ kHz) using discrete data points.

1.2 Quick Measurement of Frequency-Response Magnitude (impulse train method)

There is a quick way of getting a visual display of $|H(f)|$ as a function of f on the PicoScope (Spectrum Mode). The basic idea is that you can place a set of sinusoids, uniformly spaced in frequency and approximately equal in amplitude, on the input of the filter, and the output of the filter, in the frequency domain, will be an approximate visual representation of $|H(f)|$ versus f. In this experiment you will create a train of uniformly spaced, equal-amplitude, narrow and rectangular pulses, and this pulse train is approximately equal to the set of sinusoids needed as the input signal.

The Twin Pulse Generator will provide the pulse train. Before inserting this module into the TIMS instrument frame, look at its printed circuit board (PCB). You will see a slide switch; set it to *single* mode. You will need to provide an appropriate TTL clock signal to this module.

A clock of frequency (100/96) kHz will be used in this experiment to provide timing to the Twin Pulse Generator. Place the (100/12)-kHz TTL clock, available on the Master Signals panel (where it is called 8.3 kHz), on the input of a divide-by-four device on the Digital Utilities module. This device accepts a TTL clock and produces at its output a TTL clock having a fundamental frequency that is one-fourth that of the input. Place the output of this device on the input of a divide-by-two device (also on the Digital Utilities module). The output of the divide-by-two device is a TTL clock of fundamental frequency (100/96) kHz. This is approximately 1042 Hz. Place the (100/12)-kHz TTL clock on Channel A and the (100/96)-kHz TTL clock on Channel B. You should observe that there are 8 cycles of the former within each cycle of the latter.

Channel A: (100/12)-kHz TTL clock
Channel B: (100/96)-kHz TTL clock

Connect the (100/96)-kHz TTL clock to the TTL clock input of the Twin Pulse Generator. Every low-to-high transition occurring on this clock will generate a narrow pulse on the Twin Pulse Generator output. You can adjust the pulse width using the width knob. Rotate the width knob fully counter-clockwise, in order to produce the narrowest possible pulse. View the Q_1 output on the oscilloscope. Use the (100/96)-kHz TTL clock as an external trigger source. Measure the pulse width T_p.

Channel A: Q_1 output of Twin Pulse Generator

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Select the Spectrum Mode. Set the frequency range to 391 kHz. You should find that the spectrum consists of a set of lines, uniformly spaced by about 1042 Hz. For frequencies f within the range $0 \leq f \leq 0.4/T_p$, the lines have approximately the same height. Beyond that frequency range, the line height diminishes, and a null occurs at $f = 1/T_p$.

📊 Channel A: Q_1 output of Twin Pulse Generator

Retrieve the Tuneable LPF, still set for a bandwidth of about 5 kHz and a passband voltage gain of 1. Connect the Twin Pulse Generator Q_1 output to the filter input. View the filter output on the PicoScope (Spectrum Mode). You should see $|H(f)|$ sampled every 1024 Hz.

📊 Channel A: filter output (LPF with bandwidth 5 kHz)

In the above experimental arrangement, replace the Tuneable LPF with the 100-kHz Channel Filters module. On the front of this module select filter 2. This is a low-pass filter. (The Twin Pulse Generator Q_1 output should be connected to the input of the 100-kHz Channel Filters module.) View the filter output on the PicoScope (Spectrum Mode). You will probably want to use a frequency range of 195 kHz. Record the approximate bandwidth of this low-pass filter.

📊 Channel A: filter output (filter 2 of 100-kHz Channel Filters)

Repeat the above experiment with filter 3 of the 100-kHz Channel Filters module. This is a band-pass filter. Record the center frequency of the passband and the approximate bandwidth of this filter.

📊 Channel A: filter output (filter 3 of 100-kHz Channel Filters)

In the TIMS manual page for the 100-kHz Channel Filters module, it is stated that filter 2 is a band-pass filter and filter 3 is a low-pass filter. This is an error, as your experiment has demonstrated.

1.3 Quick Measurement of Frequency-Response Magnitude (noise method)

We can use the Noise Generator to get a quick-and-dirty visual display of $|H(f)|$ as a function of f on the PicoScope (Spectrum Mode). In this method, the Noise Generator output is connected to the filter input. The output of the Noise Generator is noise that has an approximately flat spectrum, from near DC to several hundred kilohertz. So the magnitude of the filter's transfer function will "shape" the spectrum that appears at the output of the filter.

This quick measurement should first be made with the Tuneable LPF, still set for a bandwidth of about 5 kHz and a passband voltage gain of 1.

On the Noise Generator, a noise level of $+22$ dB should be selected. The filter input should be placed on Channel A and the filter output on Channel B. Select the Spectrum Mode. Try several different values for the frequency range on the spectrum analyzer, until a nice picture of the magnitude of the transfer function is displayed.

  📊 Channel A: LPF output, showing $|H(f)|$

2 Filtering a Square-Wave

In this experiment you will see the effect of filtering a square-wave. You will use the comparator on the Utilities module as a hard limiter, to create a square-wave from a sinusoid.

sinusoid ⟶ [Hard Limiter] ⟶ square-wave ⟶ [Filter] ⟶ filtered square-wave
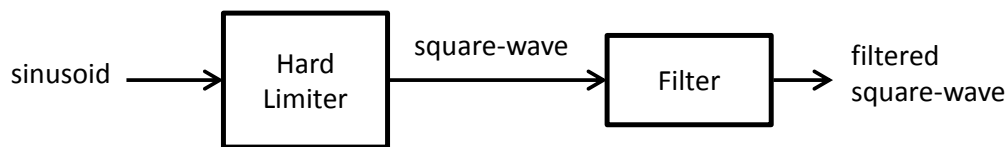
Place the 2-kHz analog sinusoid (Master Signals) on the analog input of the comparator and also on Channel A. Connect the reference input of the comparator to ground (located on the Variable DC panel). Connect the TTL output of the comparator on Channel B. Set Channel B coupling to AC, in order to block the DC component of the TTL signal. View the Channel A and B signals simultaneously. You will see that the 2-kHz sinusoid has been turned into a square-wave of the same frequency.

  〰️ Channel A: 2-kHz sinusoid
      Channel B: 2-kHz square-wave

Set the toggle switch on the front of the Tuneable LPF to *wide*. Adjust the bandwidth of the Tuneable LPF to approximately 3 kHz. (The Tuneable LPF's clock output has a frequency equal to 100 times the bandwidth.) Use the Noise Generator to get a quick measurement of $|H(f)|$.

  📊 Channel A: output of LPF (bandwidth = 3 kHz), showing $|H(f)|$

Place the TTL output of the comparator at the input of the Tuneable LPF. Connect the filter output to the RMS Meter. Adjust the passband gain so that the filter output is about 2.5 V rms.

Place a copy of the filter input on Channel A. Place a copy of the filter output on Channel B. Use AC coupling for both Channels A and B, in order to block the DC component that is present at the TTL output of the comparator.

  〰️ Channel A: input of LPF (bandwidth = 3 kHz)
      Channel B: output of LPF

📊 Channel A: input of LPF (bandwidth = 3 kHz)
　　　Channel B: output of LPF

Because the fundamental frequency is approximately 2 kHz, you should find that the 3-kHz bandwidth passes only the fundamental harmonic.

Change the bandwidth of the LPF to 7 kHz. Use the Noise Generator to get a quick measurement of $|H(f)|$.

📊 Channel A: LPF output (bandwidth = 7 kHz), showing $|H(f)|$

View the LPF output (with bandwidth = 7 kHz) on both the oscilloscope and the spectrum analyzer. Use AC coupling. You should find that the 7-kHz bandwidth passes the first and third harmonics.

〰️ Channel A: output of LPF (bandwidth = 7 kHz)

📊 Channel A: output of LPF (bandwidth = 7 kHz)

Change the bandwidth of the LPF to 11 kHz. Use the Noise Generator to get a quick measurement of $|H(f)|$.

📊 Channel A: LPF output (bandwidth = 11 kHz), showing $|H(f)|$

View the LPF output (with bandwidth = 11 kHz) on both the oscilloscope and the spectrum analyzer. Use AC coupling. You should find that the 11-kHz bandwidth passes the first, third and fifth harmonics.

〰️ Channel A: output of LPF (bandwidth = 11 kHz)

📊 Channel A: output of LPF (bandwidth = 11 kHz)

Mathematically speaking, by increasing the filter bandwidth, you are allowing more harmonics to be included in the Fourier series sum. As the bandwidth increases, the filter output resembles more closely the original square-wave at the input to the filter.

Finally, replace the Tuneable LPF with the 100-kHz Channel Filters module. Select filter 2, which is a low-pass filter with a bandwidth of about 140 kHz. The output of this filter should be a better approximation than those obtained from the output of the Tuneable LPF. View the filter output in both the time and frequency domains.

    Channel A: output of LPF (bandwidth = 140 kHz)

    Channel A: output of LPF (bandwidth = 140 kHz)

3 Phase Shifter and Hilbert Transformer

A phase shifter and a Hilbert transformer are both examples of LTI systems. For both of these systems, the interesting and important part of the frequency response is $\arg H(f)$, as the magnitude $|H(f)|$ is constant for the entire range of frequencies over which the system is intended to operate.

3.1 Phase Shifter

The Phase Shifter module has a slide switch on its PCB. Make sure this switch is set to "HI". Place the output of the Audio Oscillator at the input of the Phase Shifter, at the input of the Frequency Counter, and on Channel A. Place the output of the Phase Shifter on Channel B.

Tune the frequency to approximately 4 kHz. Adjust the delay of the Phase Shifter until the output sinusoid lags the input sinusoid by approximately $90°$.

    Channel A: input of Phase Shifter (4 kHz sinusoid)
    Channel B: output of Phase Shifter

Measure the time delay using a time ruler. Calculate the corresponding value for $\arg H(f)$. Do not make any more adjustments to the Phase Shifter for the remainder of this experiment. For each of two more frequencies, 5 kHz and 6 kHz, measure the time delay and calculate $\arg H(f)$.

    Phase Shifter

    For this Phase Shifter, plot $\arg H(f)$ as a function of f, using discrete data points.

3.2 Hilbert Transformer

The Quadrature Phase Splitter may be used as an approximation to a Hilbert transformer. A Hilbert transformer delays the phase of all sinusoids, within the design range of frequencies, by $90°$. That is, $\arg H(f) = -\pi/2$.

Place the output of the Audio Oscillator on both input 1 and input 2 of the Quadrature Phase Splitter. Also connect the Audio Oscillator to the Frequency Counter. Place output 1 of the on Channel A and the output 2 on Channel B. Tune the Audio Oscillator to a frequency of approximately 4 kHz.

Channel A: Quadrature Phase Splitter output 1 (4 kHz sinusoid)
Channel B: Quadrature Phase Splitter output 2

You should find that output 2 lags output 1 by approximately $90°$. Measure the time delay of output 2 relative to output 1. Then calculate the phase delay from $\arg H(f) = -2\pi f\tau$. Here we are regarding the $H(f)$ as the ratio of the Fourier transform of output 2 to the Fourier transform of output 1.

Hilbert transformer

You may wonder why you are measuring the delay of output 2 relative to output 1, rather than the delay of an output relative to an input. In future laboratory experiments, the Quadrature Phase Splitter will be used to provide two signals, the second of which (output 2) is the Hilbert transform of the other (output 1). In these experiments it is immaterial that there is a delay between the input 1 and output 1 (or between input 2 and output 2).

For this Hilbert transformer, plot $\arg H(f)$ as a function of f, using discrete data points.

## 4 Nonsinusoidal Signal as Input to Linear Time-Invariant Systems

You will generate a square-wave and use a Butterworth low-pass filter to select only the first and third harmonics of that square-wave. The signal at the Butterworth low-pass filter output will serve as an example of a non-sinusoidal signal and will be used as input to a Device Under Test.

The Butterworth low-pass filter that you will use is in the Baseband Channel Filters module. You will select filter 2 on that module. That filter has a bandwidth of approximately 2 kHz. Use the Noise Generator to get a quick measurement of $|H(f)|$ for that filter.

Channel A: Butterworth LPF output, showing $|H(f)|$

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You will now construct the measurement arrangement shown below. Adjust the Audio Oscillator for 580 Hz. Place the Audio Oscillator output on the analog input of the comparator, which serves here as the hard limiter. Connect the reference input of the comparator to ground. Connect the TTL output of the comparator to the input of the Butterworth low-pass filter. This TTL signal is a square-wave plus a DC component. Set the filter for AC coupling, in order to block the DC component of the TTL signal. Since the bandwidth of the Butterworth filter is approximately 2 kHz, only the fundamental and third harmonics of the 580-kHz square-wave input will pass through the filter. The fifth- and higher-order harmonics will be blocked.
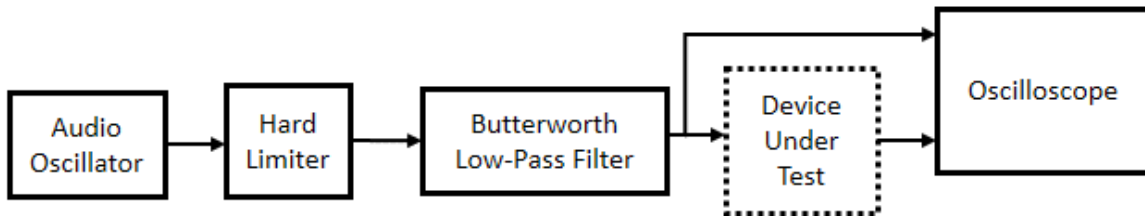
You will use the Tuneable LPF, initially, as the Device Under Test. Adjust the bandwidth of the Tuneable LPF to about 5 kHz. (The Tuneable LPF's clock output has a frequency equal to 100 times the bandwidth.) Place the output of the Butterworth low-pass filter on the input of the Tuneable LPF. Place the Tuneable LPF's input on Channel A and output on Channel B. The phase response of the Tuneable LPF is approximately linear in its passband; that is, $\arg H(f)$ is approximately proportional to f, because the delay τ is approximately constant, independent of f (within the passband). The output of the Tuneable LPF should look approximately like a delayed copy of the input.

Channel A: DUT (Tuneable LPF) input
Channel B: DUT (Tuneable LPF) output

Now you will use the Quadrature Phase Splitter as the Device Under Test. Place the output of the Butterworth low-pass filter on both inputs of the Quadrature Phase Splitter. Place output 1 of the Quadrature Phase Splitter on Channel A and output 2 on Channel B. Output 2 of this device is related to output 1 as the output of a Hilbert transformer is related to its input. That is, the phase angle $\arg H(f)$ is approximately $-\pi/2$ for each frequency (within the design limits of the device). Does output 2 look at all similar to output 1? You shouldn't expect a signal and its Hilbert transform to look alike. A Hilbert transformer is *not* a delay device.

Channel A: Quadrature Phase Splitter output 1 (Hilbert transformer input)
Channel B: Quadrature Phase Splitter output 2 (Hilbert transformer output)

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